Homework #1. Due Thursday, September 2nd, by 4pm

1. Prove the Schreier Subgroup Lemma (Theorem 1.5) without the extra assumption $1 \in T$. **Hint:** Let t_0 be the unique element of $T \cap H$. There are two places where the proof given in class needs to be modified – the initial setup of the induction and the deduction of Theorem 1.5 from the claim. In the first case write an element of G in the form $t_0 g$ and in the second case in the form $g t_0$.

2. Let F be a free group X and H a subgroup of F. Prove that H always has a Schreier transversal in F (with respect to X). Prove this in two different ways.

(a) Using Zorn's lemma

(b) Using suitable total order on F.

Hint for (b): Choose an arbitrary total order on $X \sqcup X^{-1}$ and consider the corresponding lexicographical order on F: given two elements $f \neq f' \in F$, put $f < f'$ if one of the following holds:

(i) $l(f) < l(f')$ (where $l(\cdot)$ is the word length)

(ii) $l(f) = l(f')$, and if f and f' first differ in k^{th} position, then the k^{th} symbol in f is smaller than the k^{th} symbol in f'.

Then form a transversal by choosing the smallest element in each right coset of H.

3. Let $F = F(x, y)$ be the free group on two generators. Find a free generating set for each of the following subgroups of F:

(a) $[F, F]$, the commutator subgroup of F

(b) $H = \operatorname{Ker} \pi$ where π is the epimorphism from F onto S_3 (symmetric group on 3 letters) which sends x to (12) and y to (23).

(c) Check that the element $h = xy^3x^{-1}yxy$ lies in the group H from part (b) and write h as a word in the free generators of H found in (b).

4. Use Theorem 1.5 to find a generating set with 2 elements for the alternating group A_n.