Math 8700. Lie Groups.

Problem Set 7. Due on Friday, November 1.

1. In class we proved that every torus is topologically generated by one element. Use this result to find the (minimal) number of topological generators of $\mathbb{R}^n$.

2. Let G be a compact Lie group and let L be a maximal abelian subalgebra of $\mathrm{Lie}(G)$ (so that $\exp(L)$ is a maximal torus, as we proved in class). Let $u \in L$ be such that e^u is a topological generator of $\exp(L)$. Prove that $\mathrm{Cent}_{\mathrm{Lie}(G)}(u) = L$.

3. Let G be a Lie group.

   (a) Let $u \in \mathrm{Lie}(G)$ and let ρ be a Lie algebra automorphism of $\mathrm{Lie}(G)$. Prove that $\mathrm{ad}(\rho(u)) = \rho\,\mathrm{ad}(u)\rho^{-1}$.

   (b) Let $K : \mathrm{Lie}(G) \times \mathrm{Lie}(G) \to \mathbb{R}$ the Killing form of $\mathrm{Lie}(G)$ (that is, $K(u, v) = Tr(\mathrm{ad}u\ \mathrm{ad}v)$). Use (a) to prove that K is (G, Ad)-invariant.