Homework Assignment # 10

Plan for the class on Apr 13: We will finish the discussion of cyclic extensions (end of online lecture 23) and then talk about solvable and radical extensions (online lecture 24, section 14.7 in DF).

Here and in all future assignments "online" or "online notes" refers to Algebra-II lectures posted on my Spring 2010 Algebra-II webpage

`http://people.virginia.edu/~mve2x/7752_Spring2010/`

Problems, due by 11:59pm on Friday, April 16th.

Problem 0: (practice). DF, problem 18 on p.583. **Note:** Make sure to solve this problem if you did not solve Problem 17 correctly.

Problem 1: DF, problem 21 on p.583.

Problem 2: Prove the additive form of Hilbert's Theorem 90 (DF, problem 26 on p.584). The proof is very similar to the proof of the multiplicative form we established in class.

Problem 3: DF, problems 4 and 5 on p.636.

Problem 4: Let p be a prime, $K = \mathbb{F}_p(t)$, the field of rational functions in one variable over $\mathbb{F}_p$. Let $\sigma : K \to K$ be the unique automorphism of K such that $\sigma(t) = t + 1$ and $G = \langle \sigma \rangle$ (clearly G is cyclic of order p). Let $F = K^G$. Find an explicit element s such that $F = \mathbb{F}_p(s)$ and prove your answer.

Problem 5: Let E/F be a field extension, let K_1 and K_2 be subfields of E containing F, and assume that the extensions K_1/F and K_2/F are finite. Let K_1K_2 be the composite field of K_1 and K_2. Prove that the F-algebra $K_1 \otimes_F K_2$ is a field if and only if

$$[K_1K_2 : F] = [K_1 : F][K_2 : F].$$

Hint: Use the universal property of tensor products to relate $K_1 \otimes_F K_2$ and K_1K_2.