Homework Assignment # 5.

Plan for next week: Field extensions and algebraic closures (13.1, 13.2, 13.4 and online lectures 14, 15).

Problems, to be submitted by Thu, February 28th.

Problem 1: DF, Problem 6, page 488.

Problem 2: DF, Problem 9, page 489, first two matrices are for practice.

Problem 3: Recall that for a matrix A we denoted by $\chi_A(x)$ and $\mu_A(x)$ its characteristic and minimal polynomials, respectively. Determine the number of possible RCFs of 8×8 matrices A over $\mathbb{Q}$ with $\chi_A(x) = x^8 - x^4$. Explain your argument in detail.

Problem 4: (a) Prove that two 3×3 matrices over some field F are similar if and only if they have the same minimal and characteristic polynomials. Give an example showing that this does not hold for 4×4 matrices.
(b) A matrix A is called idempotent if $A^2 = A$. Prove that two idempotent $n \times n$ matrices are similar if and only if they have they same rank. **Hint:** What is the minimal polynomial of an idempotent matrix? How does rank relate to eigenvalue 0?

Problem 5: Find the number of distinct conjugacy classes in the group $GL_3(\mathbb{Z}/2\mathbb{Z})$ and specify one element in each conjugacy class.