Homework #5.

Approximate plan for next three weeks:

Oct 11: Finitely generated abelian groups (5.2)
Oct 16, 18: Nilpotent and solvable groups (6.1)
Oct 23, 25: Free groups and presentations by generators and relations (6.3).
Note: our discussion of nilpotent groups and free groups will be significantly different from the one in Dummit and Foote.

Problems, to be submitted by 1pm on Friday, October 12th

1.

(a) Let G be an abelian group. Prove that the set of elements in G which have finite order is a subgroup of G. This subgroup is called the *torsion subgroup* of G.
(b) Give an example of a group where the set of elements of finite order is not a subgroup. **Note:** Obviously, the group has to be infinite and non-abelian by (a).

2.

(a) Prove Observation 10.1 from class: Let H, K be a group, let ϕ and ψ be homomorphisms from K to $\mathrm{Aut}(H)$, and assume that there exists $\theta \in \mathrm{Aut}(K)$ such that $\phi \circ \theta = \psi$. Prove that $H \rtimes_\phi K \cong H \rtimes_\psi K$.
(b) DF, Problem 6 on page 184.

3. DF, Problem 7(a)(c)(e) on page 185. In (e) only prove the uniqueness part (we showed the existence in class). Clarification for part (c): for each isomorphism class of S you are asked to construct certain number of non-isomorphic groups of order 56 with normal 7-Sylow and 2-Sylow isomorphic to S. You are not asked to prove that your groups cover all possible isomorphism classes (this part is optional and can be done using Problem 2(a) above). However, you should prove the statement of the hint in brackets following part (c).

4. DF, Problem 5 on page 184. The holomorph of a group G denoted by $Hol(G)$ is defined on page 179 of Dummit and Foote.

5. To be added (maybe)