**Homework #5.**

**Approximate plan for next three weeks:**

Oct 4,6: Nilpotent and solvable groups (6.1)

Oct 13: Free groups (6.3)

Oct 18,20: Free groups and presentations by generators and relations.

Note: our discussion of niplotent groups and free groups will be significantly different from the one in Dummit and Foote.

**Problems, to be submitted by Thursday, October, 6th**

**1.** Let $p$ be a prime.

(a) Let $1 \leq k < p$. Prove that a Sylow $p$-subgroup of the symmetric group $S_{pk}$ has order $p^k$ and find (some) Sylow $p$-subgroup of $S_{pk}$ explicitly.

(b) Prove that a Sylow $p$-subgroup of the symmetric group $S_{p^2}$ has order $p^{p+1}$ and find (some) Sylow $p$-subgroup of $S_{p^2}$ explicitly. **Hint:** First find a subgroup of order $p^2$ inside $S_{p^2}$ (this is done as in part (a)), call it $H$. Then find an element of order $p$ in $G \setminus H$ which normalizes $H$.

**2.** DF, Problem 6 on page 184.

**3.** DF, Problem 7(a)(c)(e) on page 185. In (e) only prove the uniqueness part (we showed the existence in class). Clarification for part (c): for each isomorphism class of $S$ you are asked to construct certain number of non-isomorphic groups of order 56 with normal 7-Sylow and 2-Sylow isomorphic to $S$. You are not asked to prove that your groups cover all possible isomorphism classes (this part is optional and can be done using a sufficient condition for two semi-direct products $H \rtimes_\phi K$ and $H \rtimes_\psi K$ to be isomorphic proved in Lecture 10). However, you should prove the statement of the hint in brackets following part (c).

**4.** DF, Problem 5 on page 184. The holomorph of a group $G$ denoted by $Hol(G)$ is defined on page 179 of Dummit and Foote.