Homework #4.

Approximate plan for next week: Direct and semi-direct products and further applications of Sylow theorems (5.4, 5.5).

Problems, to be submitted by Thursday, September 22nd

1. Let G be a finite group, P a Sylow p-subgroup of G for some p and H a subgroup of G such that $N_G(P) \subseteq H \subseteq G$. Prove that $N_H(P) = N_G(P)$ and $[G : H] \equiv 1 \mod p$.

2. Prove that a group of order $200 = 8 \cdot 25$ is not simple.

3. Prove that a group of order $132 = 3 \cdot 4 \cdot 11$ is not simple.

4. Let G be a group of order $105 = 3 \cdot 5 \cdot 7$.

(a) Prove that G has a normal Sylow 5-subgroup OR a normal Sylow 7-subgroup.

(b) Use (a) to prove that G has a normal subgroup of order 35. Deduce that G has a normal Sylow 5-subgroup AND a normal Sylow 7-subgroup.

5.

(a) Let G be an abelian group. Prove that the set of elements in G which have finite order is a subgroup of G. This subgroup is called the *torsion subgroup* of G.

(b) Give an example of a group where the set of elements of finite order is not a subgroup. **Note:** Obviously, the group has to be infinite and non-abelian by (a).