Linear algebra assessment exam. August 25th 2009.

Directions. The purpose of this test is to assess your linear algebra background and help you identify topic that you need to review. No score or grade for this test will be recored.

Solve the following problems without consulting any literature and hand in your solutions in class on Thu, August 27th.

1. Let F be a field and V a 2-dimensional vector space over F with basis $\{e_1, e_2\}$. Let $T : V \to V$ be a linear transformation whose matrix with respect to this basis is equal to $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the matrix of T with respect to the basis $\{e_1 + e_2, e_1 - e_2\}$.

2. Let U, V, W be subspaces of the same vector space. Prove or disprove the following statements:

(i) If $U \cap V = U \cap W = V \cap W = \{0\}$, the sum $U + V + W$ is direct

(ii) $(U + V) \cap W = U \cap W + V \cap W$

3. Let F be a field and $A, B \in Mat_n(F)$ be $n \times n$ matrices over F such that $AB = 0$. Prove that $rank(A) + rank(B) \le n$. **Hint:** Rank-nullity theorem.

4. Let F be a finite field of order q, let V be an n-dimensional vector space over F and $(v_1, \ldots, v_k)$ an (ordered) linearly independent subset of V. Find the number of ways to choose vectors $v_{k+1}, \ldots, v_n \in V$ such that $(v_1, \ldots, v_n)$ is a basis of V. **Hint:** First count the number of choices for v_{k+1}, then the number of choices for v_{k+2} once v_{k+1} has been chosen etc.

5. Find the Jordan canonical form of the matrix $\begin{pmatrix} 4 & 5 & 6 \\ 0 & 4 & 7 \\ 0 & 0 & 8 \end{pmatrix}$. **Hint:** It is possible to answer the question without any computations.