

## Homework #5.

### Approximate plan for next three weeks:

Oct 8: Classification of finitely generated abelian groups (5.2)

Oct 13,15: Nilpotent and solvable groups (6.1)

Oct 20,22: Free groups (6.3)

Note: our discussion of nilpotent groups and free groups will be significantly different from the one in Dummit and Foote.

### Problems, to be submitted by Friday, October, 9th

1. Let  $p$  be a prime.
  - (a) Let  $1 \leq k < p$ . Prove that a Sylow  $p$ -subgroup of the symmetric group  $S_{pk}$  has order  $p^k$  and find (some) Sylow  $p$ -subgroup of  $S_{pk}$  explicitly.
  - (b) Prove that a Sylow  $p$ -subgroup of the symmetric group  $S_{p^2}$  has order  $p^{p+1}$  and find (some) Sylow  $p$ -subgroup of  $S_{p^2}$  explicitly. **Hint:** First find a subgroup of order  $p^2$  inside  $S_{p^2}$  (this is done as in part (a)), call it  $H$ . Then find an element of order  $p$  in  $G \setminus H$  which normalizes  $H$ .
2. DF, Problem 6 on page 184. This is a generalization of Proposition 11.4 from class.
3. DF, Problem 7(a)(c)(e) on page 185. In (e) only prove the uniqueness part (we showed the existence in class).
4. DF, Problem 5 on page 184. The holomorph of a group  $G$  denoted by  $Hol(G)$  is defined on page 179 of Dummit and Foote.