Homework #6. Due Friday, March 7th, by 4pm

Reading:

1. For this homework assignment: Chapter 6.
2. For the next two classes: Chapter 6 and beginning of Chapter 7.

Problems:

1.

(a) Let $G_1, \ldots, G_k$ be finite groups. Prove that

$$\exp(G_1 \times \ldots \times G_k) = lcm(\exp(G_1), \ldots, \exp(G_k)),$$

where as usual $\exp(G)$ denotes the exponent of G.

(b) Give an example showing that if G is finite, but non-abelian, then $exp(G)$ may not equal to $o(g)$ for any $g \in G$.

2. Prove that the group U_{2^a}, with $a \geq 3$, contains precisely four elements g satisfying $g^2 = e$ and find those elements explicitly.

3. Determine whether 67 is a primitive root mod 3^{2014}.

4. Let $n = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$. Find the order of the element $[67]_n \in U_n$. **Hint:** if you use a correct approach, you can solve this problem almost without computations.

5. Find all $n \in \mathbb{N}$ for which the group U_n has exponent 4.