Homework #1. Due Thursday, September 4th, in class

Reading:

1. For this homework assignment: Chapter 1.

2. Before the class on Tue, Sep 2: finish reading Chapter 1. Before the class on Thu, Sep 4: Section 2.1 (finite, countable and uncountable sets).

Below Rudin x.y refers to Problem y after Chapter x in Rudin. In Problems 1,7 and 8 you can freely use all the standard properties of real and complex numbers.

Problems:

1. (Rudin 1.8). Prove that $\mathbb{C}$ (complex numbers) is not an ordered field.

2. Let F be an ordered field as defined in class, let P be its set of positive elements, and define the relation $<$ on F by $x < y \iff y - x \in P$. Prove that F together with relation $<$ is an ordered field in the sense of Rudin (Definition 1.17).

3. Let S be an ordered set and A and B subsets of S such that

(i) $a \leq b$ for any $a \in A$ and $b \in B$;

(ii) $\sup(A)$ and $\inf(B)$ exist in S.

Prove that $\sup(A) \leq \inf(B)$.

4. Rudin 1.5.

5. Rudin 1.9.

6. Rudin 1.20.

7. Give a detailed and rigorous proof of the fact that

$$\lim_{n\to\infty} \frac{2n+3}{3n+4} = \frac{2}{3}.$$

8. Let k be a positive integer, and let $x, y \in \mathbb{R}^k$. According to Theorem 1.37(e), we always have inequality $|x+y| \leq |x|+|y|$. Prove that equality holds if and only if $x = 0$ or $y = \lambda x$ for some scalar $\lambda \geq 0$. **Hint:** Use the proof of Theorem 1.37.