Homework #5. Due Saturday, February 24th, by 11:59pm in filedrop

All reading assignments and references to exercises, definitions etc. are from our main book 'Coding Theory: A First Course' by Ling and Xing

Reading and plan for the next week:

1. For this homework assignment read 4.8

2. Next week we will start talking about bounds for codes (Chapter 5). I am not sure about the order, but we will almost definitely discuss the sphere-covering bound (5.2) and sphere-packing bound (5.3) next week.

Problems:

1. Problem 4.41.

2. Problem 4.27. **Hint:** This problem can be solved using the same idea as 4.20(i), (ii), but there is a more conceptual solution involving cosets.

3. Problem 4.42.

4. Problem 4.47. For the decoding part use the "modified" syndrome decoding as described below. If w is the received word, compute its syndrome $S(w)$ and then find a leader of the coset $w+C$ using the result of Problem 2 (do not compute the entire coset $w+C$).

5. Problem 4.44.

6. Let $r \geq 2$, let $N = 2^r - 1$ and let $Ham(r,2)$ be the binary Hamming code of length N (see 5.3.1). Recall that we proved in Lecture 9 that $Ham(r,2)$ has distance 3.

(a) Consider the following $N+1 = 2^r$ elements: $0, e_1, \ldots, e_N$ (where e_i is the i^{th} element of the standard basis). Prove that every coset of $Ham(r,2)$ in $\mathbb{F}_2^N$ contains exactly one of these elements.

(b) Deduce from (a) that for every $w \in \mathbb{F}_2^N$ there exists $c \in Ham(r,2)$ such that $d(w,c) \leq 1$.

(c) Based on (b), what can you say about the optimized syndrome decoding (as stated at the beginning of Lecture 9 on Thu, February 15)?