Homework #6. Due Thursday, March 5th, by 1pm in my mailbox

Reading and plan for the next week:

1. For this homework assignment read 5.1-5.4.
2. Plan for next week: 5.7 (Griesmer bound) and 5.5 (Plotkin bound), in this order; perhaps a bit more on MDS codes (5.4). If time left, we may talk about Reed-Muller codes (6.2).

Problems:

1. Problem 5.3(ii). **Hint:** PCM in the special case $\varepsilon = 1$ is given by Theorem 5.1.9 (note that this theorem is NOT proved in the book). To justify your answer for PCM it may be convenient to use Theorem 5.2 from class.

2. Problem 5.7.

3.

(a) Show that the all-one vector $(1, 1, \dots, 1)$ of length 24 lies in the extended binary Golay code G_{24}.

(b) Assume without proof that G_{24} contains (exactly) 759 words of weight 8. Use this fact and (a) to prove that the distribution of weights in G_{24} is given by Table 5.5 on page 109.

4.

(a) Prove that the Golay code G_{23} is perfect by checking that it matches the sphere-packing bound.

(b) Use Problem 3(b) to prove that possible weights of elements of G_{23} are $0, 7, 8, 11, 12, 15, 16$ and 23. Make sure to prove that each of those numbers actually arises as the weight of some element of G_{23}.

(c) Let $w \in \mathbb{F}_2^{23}$ with $wt(w) = 4$. Let $c_w \in G_{23}$ be the result of applying NND decoding (with respect to G_{23}) to w. Use (a) and (b) to prove that $d(w, c_w) = 3$ and $wt(c_w) = 7$.

5. Problem 5.19. **Note:** Simplex codes $S(r, q)$ are defined at the end of 5.3.2, page 88.

6. Let $r \geq 2$ be an integer.

(a) Assume that $d \geq 3$. Prove that there is no binary $[2^r, 2^r - r, d]$-linear code.

(b) Given an example of a binary $[2^r, 2^r - r - 1, 4]$-linear code.