Homework #3. Due Wednesday, February 5th, in class

All reading assignments and references to exercises, definitions etc. are from our main book 'Coding Theory: A First Course' by Ling and Xing

Reading and plan for the next week:

1. For this homework assignment read 4.2-4.6

2. Next week we will continue with the basic theory of linear codes (4.2-4.6). Then (probably next Wednesday) we will introduce binary Hamming codes (5.3.1, page 84). If there will be time left, we will go back to Chapter 4 and start discussing encoding and decoding for linear codes (4.7-4.8).

Problems:

1. Problem 4.3. **Hint:** First count the number of ordered k-tuples $(v_1, \ldots, v_k)$ such that the vectors $v_1, \ldots, v_k$ are linearly independent. This can be done by using the argument from the proof of Theorem 4.1.15(ii).

2. Problem 4.14. If the code is linear, find its generator matrix and its parity-check matrix.

3. Problem 4.15.

4. Problem 4.20.

5.

(a) Let C be a binary linear code of length n. Fix $1 \leq i \leq n$, consider the i^{th} coordinates of all codewords in C. Prove that either all codewords have 0 as their i^{th} coordinate or exactly half of all codewords have 0 as their i^{th} coordinate. This is based on the same idea as 4.20(iii).

(b) In Homework 1 we proved that if C is a binary code of length n and distance 2, then $|C| \leq 2^{n-1}$; thus, if in addition C is linear, then $\dim C \leq n-1$. Now prove that if C is a binary $[n, n-1, 2]$-linear code, then C is the parity-check code. **Hint:** Use induction on n and part (a). If you need a more detailed hint, see next page.

6. Problem 4.22.

7. Problem 4.31.

Hint for 5(b): For the induction step take an arbitrary binary $[n, n-1, 2]$-linear code C, consider the set $C' = \{w \in \mathbb{F}_2^{n-1} : w0 \in C\}$ (here $w0$ is the concatenation of w and 0), show that C' is a linear code of Hamming distance ≥ 2 (and length n) and apply part (a).