**Math 4310. Fall 2018. Extra practice problems the first midterm.**

**1.** Let $X$ be a metric space.

(a) Prove that if $X$ is covering compact, then $X$ is bounded directly from the definition of covering compactness.

(a) Assume that $X$ is sequentially compact and $Y$ is a closed subset of $X$. Prove that $Y$ is sequentially compact directly from the definition of sequential compactness.

**2.** Let $X$ be a complete metric space. Prove that any closed subset of $X$ is complete.

**3.** Prove Lemma 10.6 from class; if $\{x_n\}$ is a Cauchy sequence in some metric space $X$, and $\{x_n\}$ contains a convergent (in $X$) subsequence, then $\{x_n\}$ converges in $X$. **Hint:** One can give a very short proof using the existence of completion of $X$ (really all you need to know is that $X$ is a subset of some complete metric space $\widehat{X}$).

Problems from Pugh:

(a) Problem 40 after Chapter 1

(b) Problems 7, 13, 14, 22, 28(a)-(d), 44, 55(a), 96, 108, 116, 125, 140 after Chapter 2.