Math 3354. Spring 2016. Brief study guide for the second midterm.

The second midterm will be given in class on Thursday, April 14th, in class. It will cover material from Lectures 10-18(the corresponding online lectures are 10-18 and 16A). The relevant book material is covered in

> Pinter: Chapters 3-5, 7-11, 13 (only the statement and applications of Lagrange Theorem) and 14.
> Gilbert: entire Chapter 3, 4.1, 4.2 and parts of 4.4 (only the statement and applications of Lagrange Theorem)

Below is the main list of topics/standard type of problems you need to know; for each item I list the corresponding lecture (using online numbering) and the most relevant homework problems.

(1) Definition of a group. Proving that something as a group. Lecture 10. HW 6.1-6.3
(2) Basic examples of groups and group constructions (including direct products). Most of these examples are introduced in Lecture 10; see also the end of Lecture 18.
(3) Basic properties of group elements. Lecture 11. HW 6.5-6.7
(4) Subgroups. Basic methods of constructing subgroups. Proving that something is a subgroup. Lecture 12. HW 6.8-6.11, 7.7(a)
(5) Orders of elements. Computing the order of elements (both in concrete groups and by an abstract argument). Formulas for the orders of elements in cyclic groups and in symmetric groups. Lecture 13 and Lecture 17.3. HW 7.1, 7.2, 7.9, 7.10, 8.6, 8.7, 9.4, 9.5
(6) Structure theorem of finite cyclic groups (you need to know at least parts (i), (ii), (iii) and (v) of Theorem 14.1). Lecture 14.1. HW 7.2, 7.3, 7.4, 8.4(c)
(7) Isomorphisms. How to prove that two groups are isomorphic. Lectures 14.2 and 15 (up to and including example 3). HW 7.6, 7.7(b), 8.7
(8) How to prove that two groups are not isomorphic. The end of Lecture 15. HW 7.11, 8.7
(9) Homomorphisms. Important examples of homomorphisms. How to prove that a map is a homomorphism. First page of Lecture 16. HW 8.1(a), 8.2(a)(b), 8.3, 8.4
(10) Basic theorems about homomorphisms and their applications. The rest of Lecture 16. HW 8.1(b), 8.2, 9.6
(11) Classification of finite abelian groups. Lecture 16A. HW 8.7, 8.8.
(12) Symmetric groups. Lecture 17.1-17.3. HW 9.1-9.5
(13) Lagrange Theorem (statement and applications). Lecture 18. HW 9.6-9.8
(14) Cayley's Theorem. You do not need to know the full proof, but you should know the construction of an injective homomorphism $\phi : G \to S_n$ in the proof of Cayley's theorem and why the existence of such homomorphism proves the statement of the theorem. Lecture 17.4. HW 9.9.