Math 3354. Spring 2016. Brief study guide for the final.

The final will be given on Friday, May 6, 9am-12:10pm, in our usual classroom Monroe 124. It will cover material from class lectures 4-25 (the corresponding online lectures are 4-26, 16A and the note on even/odd permutations). Last year's final exam should give you a general idea on how many questions from each part of the course (before the first midterm, between the midterms and after the second midterm) you should expect.

New this year: Most likely one of the problems on the final will ask for a proof of one or more of the results proved in class. You may be asked to state and prove any theorem/lemma/proposition etc. satisfying the following two conditions:

(i) The result has a specific name (rather than just a number) in the online notes, e.g. Lagrange Theorem
(ii) The proof in the online notes is at most one page long.

You do not have to worry about the results for which complete proof is not given in the online notes.

Below is the main list of topics/standard type of problems you need to know; for each item I list the corresponding lecture (using online numbering) and the most relevant homework problems. Items 5-18 are taken directly from the study guide for the second midterm.

(1) Divisibility and Greatest Common Divisor. Lecture 4. HW 3.1-3.5
(2) Primes and Unique Factorization Theorem. Lecture 5. HW 3.6, 3.7
(3) Congruences and the Chinese Remainder Theorem. Lectures 6-7. HW 4.1, 4.2, 4.4-4.6
(4) Rings of congruences classes $\mathbb{Z}_n$. Lectures 8-9. HW 5.7, 5.8, 7.1, 8.3-8.6, 11.2, 11.5(a)(b)
(5) Definition of a group. Proving that something as a group. Lecture 10. HW 6.1-6.3
(6) Basic examples of groups and group constructions (including direct products). Most of these examples are introduced in Lecture 10; see also the end of Lecture 18.
(7) Basic properties of group elements. Lecture 11. HW 6.5-6.7
(8) Subgroups. Basic methods of constructing subgroups. Proving that something is a subgroup. Lecture 12. HW 6.8-6.11, 7.7(a)
(9) Orders of elements. Computing the order of elements (both in concrete groups and by an abstract argument). Formulas for the orders of elements in cyclic groups and in symmetric groups. Lecture 13 and Lecture 17.3. HW 7.1, 7.2, 7.9, 7.10, 8.6, 8.7, 9.4, 9.5
(10) Structure theorem of finite cyclic groups (you need to know at least parts (i), (ii), (iii) and (v) of Theorem 14.1). Lecture 14.1. HW 7.2, 7.3, 7.4, 8.4(c)
(11) Isomorphisms. How to prove that two groups are isomorphic. Lectures 14.2 and 15 (up to and including example 3). HW 7.6, 7.7(b), 8.7
(12) How to prove that two groups are not isomorphic. The end of Lecture 15. HW 7.11, 8.7
(13) Homomorphisms. Important examples of homomorphisms. How to prove that a map is a homomorphism. First page of Lecture 16. HW 8.1(a), 8.2(a)(b), 8.3, 8.4

(14) Basic theorems about homomorphisms and their applications. The rest of Lecture 16. HW 8.1(b), 8.2, 9.6
(15) Classification of finite abelian groups. Lecture 16A. HW 8.7, 8.8.
(16) Symmetric groups. Lecture 17.1-17.3. HW 9.1-9.5
(17) Lagrange Theorem (statement and applications). Lecture 18. HW 9.6-9.8
(18) Cayley's Theorem. You do not need to know the full proof, but you should know the construction of an injective homomorphism $\phi : G \to S_n$ in the proof of Cayley's theorem and why the existence of such homomorphism proves the statement of the theorem. Lecture 17.4. HW 9.9.
(19) Cosets and the proof of Lagrange Theorem. Lecture 19. HW 10.1, 10.2,
(20) Normal subgroups. Main examples of normal subgroups. Conjugation criterion. Lecture 20. HW 10.3, 10.4.
(21) Conjugacy classes. Connections with normal subgroups and centralizers. Conjugacy classes in symmetric groups. Lecture 21. HW 10.5, 10.7, 11.6, 11.7
(22) Even and odd permutations. Online note. HW 10.6
(23) Quotients Groups and FTH. Lectures 22-23. HW 11.1-11.5
(24) Rings and subrings. Lecture 24. HW 12.1, 12.2
(25) Ideals, quotients rings, ring homomorphisms and ring-theoretic FTH. Lectures 25-26. HW 12.3-12.5