Homework #12. Not due

Reading:

For this assignment: 6.1-6.3 + class notes (Lectures 24-25).

Problems:

Problem 1: Fill in the details of the proof of comparison test for series (an outline of proof was given in Lecture 24).

Problem 2: Let $\sum a_n$ and $\sum b_n$ be convergent series.

(a) Suppose that $a_n \geq 0$ and $b_n \geq 0$. Prove that the series $\sum a_n b_n$ also converges. **Hint:** Use comparison test.

(b) Show that if we do not assume that $a_n \geq 0$ and $b_n \geq 0$, then the series $\sum a_n b_n$ may diverge.

Problem 3: Let $\{a_n\}_{n=1}^{\infty}$ be a sequence, and define the sequence $\{b_n\}$ by $b_n = a_{2n-1} + a_{2n}$.

(a) Prove that if $\sum a_n$ converges, then $\sum b_n$ also converges. **Hint:** Reformulate the problem in terms of sequences of partial sums.

(b) Assume now that $\sum b_n$ also converges and $a_n \to 0$. Prove that $\sum a_n$ converges. **Hint:** one way to prove this is to apply the result of HW#3.7 to the sequence of partial sums of $\sum a_n$.

(c) Show that without the assumption $a_n \to 0$ it is possible that $\sum b_n$ converges but $\sum a_n$ diverges.

Problem 4: Find all $x \in \mathbb{R}$ for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges. **Note:** for most values of x, the answer is given by the ratio test as stated in the book (see Theorem 6.24).