Homework #9, not due

Note: This homework is not due; however, Quiz #4 on Thu, Apr 7, will be based on this assignment.

Reading:

1. For this assignment: Lectures 17 and 18 (March 29,31) and BT, §3.4 and parts of 4.1 (Examples 4.8 and 4.9 are the most relevant for this assignment)

2. For next week's classes: BT, parts of 3.5 dealing with conditional PDFs (pp. 169-172) and 4.2.

Problems:

Problem 1: Let B be the upper-half of the unit disc of radius 1 centered at $(0,0)$, that is, $B = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ and } y \geq 0\}$.

(a) Let (X,Y) be a pair of random variables uniformly distributed on B. Compute the marginal PDFs f_X and f_Y (first you need to write down an explicit formula for $f_{X,Y}$).

(b) Now let (X,Y) be a jointly continuous pair of random variables with joint PDF $f_{X,Y}$ given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{4}{\pi}(x^2+y^2) & \text{if } (x,y) \in B \\ 0 & \text{if } (x,y) \notin B \end{cases}$$

Again compute f_X and f_Y.

Problem 2: Let (X,Y) be a jointly continuous pair of random variables with joint PDF $f_{X,Y}$ given by

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+y)} & \text{if } 0 \leq x \leq y \\ 0 & \text{otherwise} \end{cases}$$

(This is a legitimate joint PDF by Problem 5 in HW#8). Find the formula for the associated joint CDF $F_{X,Y}$.

Hint: For notational purposes it is convenient to denote the input variables of $F_{X,Y}$ by a and b (rather than x and y) in which case the formula for CDF in terms of PDF can be rewritten as follows:

$$F_{X,Y}(a,b) = \int_{-\infty}^{a} \left(\int_{-\infty}^{b} f_{X,Y}(x,y)dy \right) dx.$$

Consider three cases: $a < 0$ or $b < 0$ (case 1); $0 \leq b < a$ (case 2) and $0 \leq a \leq b$ (case 3). Draw the picture! (different picture in each case). Recall that geometrically $F_{X,Y}(a,b)$ represents the double integral of the joint PDF over the region $\{(x,y) \in \mathbb{R}^2 : x \leq a, y \leq b\}$. In each case integration should reduce to computing the integral of $2e^{-(x+y)}$ over some bounded region.

Problem 3: Let T be the triangle in $\mathbb{R}^2$ with vertices $(0,0)$, $(1,0)$ and $(1,1)$. Let (X,Y) be a jointly continuous pair of random variables with joint PDF $f_{X,Y}$ given by

$$f_{X,Y}(x,y) = \begin{cases} x+4y & \text{if } (x,y) \in T \\ 0 & \text{if } (x,y) \notin T \end{cases}$$

(This is a legitimate joint PDF as verified in Lecture 17). Let $Z = X + Y$. Compute the PDF and CDF of $X + Y$.

Hint: We will denote the input variable of the CDF by z. By definition $F_Z(z) = P(X + Y \leq z)$. In Lecture 17 we expressed $F_Z(1)$ as the integral of $x + 4y$ over certain region. The same can be done for $F_Z(z)$ for all z, but the shape of the region will be different in the following four cases: $z < 0$, $0 \leq z \leq 1$, $1 \leq z \leq 2$, $z > 2$. Once F_Z is found, f_Z can be computed by differentiation.

Problem 4: Problem 5 at the end of Chapter 4 in BT.

Problem 5: Problem 6 at the end of Chapter 4 in BT.

Problem 6: Problem 7 at the end of Chapter 4 in BT.