Homework #12, due on Thursday, April 28

Reading and plan for next week:

1. For this assignment: §5.1 and 5.4.

2. For next week's classes: §3.3 and Examples 4.5 and 4.11 in §4.1. The plan for next week is to do an additional example on CLT and then discuss the geometric interpretation of CLT. After that we will talk about normal random variables in more detail; in particular, we will prove that a linear function of a random variable is normal (Example 4.5 in §4.1) and that the sum of normal random variables is normal (Example 4.11 in §4.1)

Problems:

Problem 1: Suppose we roll a fair die 54 times, and let X be the number of times we roll 3. Prove that

$$P(X \leq 18) \geq 0.925.$$

This is the same question as Problem#2 on quiz#5 except that you are asked to prove a stronger inequality.

Note: In Problems 2-7 use normal approximation based on CLT. Use histogram correction in all problems except #6.

Problem 2: (Durrett 6.24) In a 162 game season find the approximate probability that a team with a 0.5 chance of winning will win at least 87 games.

Problem 3: (Durrett 6.28) Suppose we roll a die 600 times. What is the approximate probability that the number of 1's obtained lies between 90 and 110 (including 90 and 110)?

Problem 4: (Durrett 6.33) A gymnast has a difficult trick with a 10% chance of success. She tries the trick 25 times and wants to know the probability she will get exactly two successes. Compute the (a) exact answer, (b) Poisson approximation, (c) normal approximation.

Problem 5: (Durrett 6.34) A student is taking a true/false test with 48 questions.

(a) Suppose she has a probability $p = \frac{3}{4}$ of getting each question right. What is the probability she will get at least 38 right?

(b) Answer the last question if she knows the answers to half the questions and flips a coin to answer the other half.

Notice that in each case the expected number of questions she gets right is 36.

Problem 6: (Durrett 6.39) An electronics company produces devices that work properly 95% of the time. The new devices are shipped in boxes of 400. The company wants to guarantee that k or more devices per box work. What is the largest k so that at least 95% of the boxes meet the warranty? **Hint:** If you are not sure about the initial setup for this problem, see solutions to Problem 6 in HW#6 (this problem is quite similar except that you should use normal approximation instead of Poisson approximation).

Problem 7: (Durrett 6.42) Suppose we roll a die 10 times. What is the approximate probability that the sum of the numbers obtained lies between 30 and 40 (including 30 and 40)?