Homework #11.

This assignment is not due. However quiz #5 on Thursday, April 21, will be based on this assignment.

Reading:

1. For this assignment: Lectures 20-22 (April 7,12,14) and BT, §4.2, §5.1 and §5.2.

2. For next week's classes: BT, §5.4; also review §3.3 the procedure of standardizing a normal random variable (pp. 157 in BT)

Problems:

Problem 1-3: BT, Problems 17,18 and 19 after chapter 4.

Problem 4: Suppose we roll a fair die 24 times. Let

X be the number of times we get 1 on rolls 1-12,
Y the number of times we get 1 on rolls 13-24,
Z the number of times we get 2 on rolls 1-12 and
W the number of times we get 1 on rolls 12-24.

In Lecture 21 (on April 12) we computed the covariances $Cov(X, Y)$, $Cov(X, Z)$ and $Cov(X, W)$ and the corresponding correlation coefficients. Compute the remaining covariances $Cov(Y, Z)$, $Cov(Y, W)$ and $Cov(Z, W)$ and the corresponding correlation coefficients using similar techniques.

Problem 5: Suppose we toss a fair coin n times and let X be the number of heads. Find the smallest n for which the Chebyshev inequality guarantees that $P(|\frac{X}{n} - \frac{1}{2}| \geq 0.01) \leq 0.001$.

Problem 6: You are waiting in a line at a supermarket and find 9 customers ahead of you. The service times (in minutes) for each customer are exponentially distributed with $\lambda = \frac{1}{2}$ and independent from each other. Use the Chebyshev inequality to find the probability that your total waiting time (including your service time) is less than 40 minutes.

Problem 7:

(a) A cricket is jumping along a straight line in the same direction. The length of its jump is uniformly distributed (continuously) between 8 and 12 cm. Let X be the total distance by which the cricket moves after 10 jumps. Use the Chebyshev inequality to estimate $P(|X - 100| < 10)$.

(b) Now suppose that each time the cricket is equally likely to jump forward or backwards, with jump length still uniformly distributed between 8 and 12 cm. Let $X(n)$ be the position of the cricket after n jumps (we assume that the initial position is at 0). Find the smallest n for which the Chebyshev inequality guarantees that $P(|X(n)| \geq n) \leq \frac{1}{3}$.

Note: To solve (a) you need to use the formula for the mean and variance of a uniform random variable derived in [BT, pp.145-146]. For (b) you need to derive similar formulas for the random variable measuring the change of the cricket position after one jump (taking direction of the jump into account).