Math 3000 assessment exam

This exam will test your familiarity with basic properties of sets and functions as well as basic proof techniques (contradiction and induction). The goal of this exam is to help you decide whether you should take Math 3000 Transition to Higher Mathematics prior to/concurrently with Math 3310 and Math 3354. Math 3000 will be offered in Fall 2016, MWF 11-11:50am.

- You do not need to take this exam if you do not plan to major or minor in math or if you already completed (or will be completing this term) both Math 3310 and Math 3354.
- The general guidelines regarding the exam are as follows. If you have no idea how to solve most of the problems, taking Math 3000 is definitely a good idea. If you know how to solve all the problems and confident that your answers are correct, you do not need to take Math 3000. If you are somewhere in between, you should complete the test (in writing) and show it to one of your instructors, your math advisor (if you have one) or anyone else in the math department you feel comfortable talking to, in order to get a recommendation regarding Math 3000.
- Note that it is not assumed that students know how to solve problems on this exam prior to enrolling in Math 3310 and Math 3354, and it is possible to pick up this material while taking Math 3310/3354; however, in Math 3000 you will have an opportunity to learn this material more thoroughly and at a more comfortable pace.

1. Prove by contradiction that $\sqrt{12}$ is irrational.

2. Define the sequence $\{a_n\}$ by $a_1 = 4$ and $a_n = 2a_{n-1} - 3$ for all $n \geq 2$. Use mathematical induction to prove that $a_n = 2^{n-1} + 3$ for all n.

3. For each of the following functions determine whether it is injective (one-to-one) and whether it is surjective (onto). Here $\mathbb{N}$ stands for natural numbers (=positive integers), $\mathbb{R}$ for real numbers and $\mathbb{R}_{\geq 0}$ for non-negative real numbers. Justify your answers.

(a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x$
(b) $f : \mathbb{N} \to \mathbb{N}$ given by $f(x) = 2x$
(c) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$
(d) $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ given by $f(x) = x^2$

Problem **4** is on the back.

4. Let $f : X \to Y$ be a function. Given a subset A of X, let

$$f(A) = \{f(x) : x \in A\}$$

be the image of A under f (this is a subset of Y). Given a subset B of Y, let

$$f^{-1}(B) = \{x \in X : f(x) \in B\}$$

be the preimage (=inverse image) of B under f, that is, $f^{-1}(B)$ is the set of all elements of X which get mapped to an element of B under f.
For each of the following statements determine whether it is true (for all possible functions) or false (for at least one function). If the statement is true, prove it; if it is false, give a specific counterexample.

(i) If B is a subset of Y, then $f(f^{-1}(B)) \subseteq B$
(ii) If B is a subset of Y, then $f(f^{-1}(B)) \supseteq B$
(iii) If A and C are subsets of X, then $f(A \cap C) = f(A) \cap f(C)$
(iv) If B and D are subsets of Y, then $f^{-1}(B \cap D) = f^{-1}(B) \cap f^{-1}(D)$