Homework #9. Due on Friday, April 6th by 1pm in TA's mailbox

Reading:

1. For this assignment: sections 5.2 and 6.1 of the BOOK.
2. For next week's classes: sections 6.1 and 6.2.

Practice problems from the BOOK: From 5.2: 3(a)(c),8,9; from 6.1: 2,4,8,10.

Problems to hand in:

1. Problem 3(b)(d) from Section 5.2 (make sure to prove your answer)

2. Define a relation $\sim$ on $\mathbb{Z}$ by $x \sim y \iff x^2 \equiv y^2 \mod 5$ (that is, $x \sim y \iff 5 \mid (y^2 - x^2)$). Prove that $\sim$ is an equivalence relation and describe the equivalence classes with respect to $\sim$: find the number of distinct equivalence classes and explicitly describe the elements in each class.

3. For each of the following functions determine whether it is injective and whether it is surjective (include detailed justifications):

(a) $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$
(b) $f : \mathbb{R} \to \mathbb{R}_{\geq 0}$ given by $f(x) = x^2$
(c) $f : \mathbb{Q}_{\geq 0} \to \mathbb{Q}_{\geq 0}$ given by $f(x) = x^2$
(d) $f : \mathbb{R}_{\geq 0} \to [0, 1)$ given by $f(x) = \frac{x}{x+1}$. Here $[0, 1)$ is the half-open interval $\{x \in \mathbb{R} \mid 0 \leq x < 1\}$.

4. Problem 5 in Section 6.1 (make sure to prove your answer)

5. Problem 12 in Section 6.1.

6. Let $f : A \to B$ be a function. Define a relation $\sim_f$ on A by $a \sim_f b \iff f(a) = f(b)$.

(a) Prove that $\sim_f$ is an equivalence relation
(b) Fix an integers $n \geq 2$, let $A = \mathbb{Z}$ and $B = \{0, 1, \ldots, n-1\}$. Construct a function $f : A \to B$ such that the equivalence classes with respect to the relation $\sim_f$ defined above are precisely the congruence classes mod n (that is, the equivalence classes with respect to $\equiv \mod n$). Your f should be given by a simple verbal description.

7. Let A and B be non-empty finite sets, $n = |A|$ and $m = |B|$. Use the fundamental principle of counting to prove that

(a) The total number of functions from A to B is equal to m^n.
(b) The total number of injective functions from A to B is equal to $m(m-1)\ldots(m-n+1)$ if $m \geq n$ and is equal to 0 if $m > n$.