**Homework #7. Due on Friday, March 23rd by 1pm in TA's mailbox**

**Reading:**

1. For this assignment: sections 4.1-4.3 of the BOOK (throughout this assignment BOOK refers to the book 'A discrete transition to advanced mathematics').
2. For next week's classes: sections 5.1, 5.2 + additional sections TBA.

**Practice problems from the BOOK:** From 4.1: 4, 7, 8; from 4.2: 5,8; from 4.3: 5,8.

**Problems to hand in:**

**1.** Compute the first 10 rows of the Pascal triangle.

**2.** Prove the binomial theorem by induction on $n$ using Lemma 13.4. **Hint:** Do the induction step in the form $S_{n-1} \Rightarrow S_n$ (or replace $n$ by $n+1$ in Lemma 13.4). Also, it is probably technically easier to work with the expanded expression in the binomial theorem rather than working with the $\Sigma$-notation.

**3.** Prove the equality $\binom{n}{k} = \binom{n}{n-k}$ using either the definition of binomial coefficients or the binomial theorem. You are not allowed to use the explicit formula for binomial coefficients involving factorials.

**4.** Let $A$ be a non-empty finite set. Prove that the total number of subsets of $A$ which have even cardinality is equal to the total number of subsets of $A$ which have odd cardinality. **Hint:** It is enough to do the case $A = [n]$. Express both numbers in question in terms of binomial coefficients and use the binomial theorem.

**5.** Let $n \in \mathbb{N}$, and write $n = p_1^{a_1} \dots p_k^{a_k}$ where $p_1, \dots, p_k$ are distinct primes and $a_1, \dots, a_k \in \mathbb{N}$. Use the general Fundamental Principle of Counting (FPC) to show that the number of positive divisors of $n$ is equal to $\prod_{i=1}^{k}(a_i+1)$. Give a detailed argument.

**6.** Let $n \in \mathbb{N}$. Use Problem 5 and a suitable result from HW#6 to prove that the number of positive divisors of $n$ is odd if and only if $n$ is a perfect square.

**7.** Problem 24 in Section 3.1. **Hint:** There is a reason why this problem appears in this homework.

**8.** Problem 7 in Section 4.2. Make sure to prove your answer.